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C.B.S. (Eighth Semester)
EXAMINATION, May - June, 2022
MATHEMATICS STREAM
FOURIER ANALYSIS
(M-801)

Time : Three Hours] *[Maximum Marks:40*

Note : Attempt all sections as directed.

(Section-A)

(0.5 marks each)

Choose the correct/most appropriate answer and write in your answer book:

1. If a periodic function *f(x)* is ___________, its fourier expansion contains only cosine terms.

   (A) Odd

   (B) Even

   (C) Multiple of 3

   (D) Multiple of 5

2. In fourier series, the function *f(x)* is ___________

   (A) Multivalued

   (B) Infinite

   (C) Finite

   (D) Non-periodic

3. Fourier series of a discontinuous function is not ________ at all points.

   (A) devergent

   (B) Convergent

   (C) Uniformly Convergent

   (D) None of the above

4. A function *f(x)* is said to be even (or symmetric) function if,

   (A) $f(-x) = f(x)$

   (B) $f(-x) = f\left(\frac{x}{2}\right)$

   (C) $f(-x) = f\left(\frac{-x}{2}\right)$

   (D) $f(-x) = f\left(\frac{-x}{4}\right)$

5. A trigonometric series $\sum_{n=-\infty}^{n=\infty} c_n e^{inx}$ ______________ if the limit $\lim_{N\to\infty} \sum_{|n|\le N} |c_n|$ exists.

(A) Converges pointwise

(B) Converges uniformly

(C) Converges absolutely

(D) Converges

6. The function $\mathrm{P_r}(x) = \dfrac{1-r^2}{1-2r\cos x + r^2}$ is called _______

(A) Parseval Equality

(B) Conjugate Poisson Kernel

(C) Poisson Kernel

(D) Fejer's Means

7. The Direchlet kernel satisfies _____________

(A) $D_N(0)=2N$

(B) $D_N(0)=2N+1$

(C) $D_N(0)=N+1$

(D) $D_N(0)=3N+1$

8. Let 0<a<b<1, let *f* be an integrable function on [*a*,*b*]. Then

(A) $\lim_{N\to\infty} \int_a^b f(t)D_N(t)dt = 0$

(B) $\lim_{N\to\infty} \int_a^b f(t)D_N(t)dt = -1$

(C) $\lim_{N\to\infty} \int_a^b f(t)D_N(t)dt = 1$

(D) $\lim_{N\to\infty} \int_a^b f(t)D_N(t)dt = ½$

9. If $f(x) = e^{-\pi x^2}$ then __________

(A) $\hat{f}(\xi) = f(\xi)$

(B) $\hat{f}(\xi) = 3f(\xi)$

(C) $\hat{f}(\xi) = 5f(\xi)$

(D) $\hat{f}(\xi) = 2f(\xi)$

10. If *F(s)* is the complex fourier transform of *f(x)*, then

(A) $F\{f(x-a)\} = e^{isa}F(s)$

(B) $F\{f(x-a)\} = e^{-isa}F(s)$

(C) $F\{f(x+a)\} = e^{isa}F(s)$

(D) $F\{f(x+a)\} = e^{-isa}F(s)$

11. The Fourier sine integral of $f(x)$ is given by __________

(A) $f(x)=\frac{2}{\pi}\int_0^\infty \sin\lambda x\int_0^\infty f(t)\sin\lambda t dt d\lambda$

(B) $f(x)=\frac{1}{\pi}\int_0^\infty \sin\lambda x\int_0^\infty f(t)\sin\lambda t dt d\lambda$

(C) $f(x)=\frac{1}{\pi}\int_0^\infty \sin\lambda x\int_0^\infty f(t)\cos\lambda t dt d\lambda$

(D) $f(x)=\frac{1}{\pi}\int_0^\infty \cos\lambda x\int_0^\infty f(t)\cos\lambda t dt d\lambda$

12. The Fourier cosine transform of e^{-x^2} is given by _____

(A) $F_c(e^{-x^2})=\frac{\sqrt{\pi}}{2}e^{-s^2/4}$

(B) $F_c(e^{-x^2})=\frac{\sqrt{\pi}}{2}e^{-s^2/2}$

(C) $F_c(e^{-x^2})=\frac{\sqrt{\pi}}{2}e^{-s^2/3}$

(D) $F_c(e^{-x^2})=\frac{\sqrt{\pi}}{4}e^{-s^2/2}$

13. $e^{-|x|}$ decreases rapidly at _____, it is not differential at 0 and therefore does not belong to S(R).

(A) 0.5

(B) Infinity

(C) zero

(D) -0.5

14. Which one is correct?

(A) Schwartz space is not closed under differentiation and multiplication by polynomials

(B) If $f \in S(R)$ then $\hat{f} \notin S(R)$

(C) Schwartz space is closed under differentiation and multiplication by ploynomials

(D) $\hat{f}$ is not bounded and not continous

15. A function f defined on R is said to be of _____ if f is continuous and there exists a constant A>0 so that

$$|f(x)| \le \frac{A}{1+x^2}\ for\ all\ x \in R$$

(A) Moderate Increase

(B) Moderate Constant

(C) Moderate Decrease

(D) None of the above

16. If $f, g \in S(R)$ then __________

(A) $f * g = g * f$

(B) $f * g \neq g * f$

(C) $f * g = \frac{g}{2} * f$

(D) $f * g = \frac{g}{4} * f$

17. Let $\langle T_n \rangle$ be a sequences in S*. We say that $\langle T_n \rangle$ _______ to T in S* iff $\lim_{n\to\infty} \langle T_n, \varphi \rangle = \langle T, \varphi \rangle$ for every $\varphi \in S$

(A) Uniformly converges

(B) Diverges

(C) Converges

(D) Pointwise converges

18. The $\propto$th distributional derivative of T is the tempered distribution $\partial^{\propto}$T defined by _______

(A) $\langle \partial^{\propto} T, \varphi \rangle = (-1) \langle T, \partial^{\propto} \varphi \rangle \; for\, all\, \varphi \in S$

(B) $\langle \partial^{\propto} T, \varphi \rangle = \langle T, \partial^{\propto} \varphi \rangle \; for\, all\, \varphi \in S$

(C) $\langle \partial^{\propto} T, \varphi \rangle = (-1)^{|\propto|} \langle T, \partial^{\propto} \varphi \rangle \; for\, all\, \varphi \in S$

(D) $\langle \partial^{\propto} T, \varphi \rangle = (-2) \langle T, \partial^{\propto} \varphi \rangle \; for\, all\, \varphi \in S$

P.T.O.

19. The function $\left(\frac{1}{x}\right)$:$R/0 \rightarrow$ R has ____________at x=0, so it does not define a regular distribution.

(A) Singularity

(B) Integrable singularity

(C) Non-integrable singularity

(D) None of the above

20. The topological dual space of S, denoted by S* or S' is the space of continuous linear functionals $T{:}S \rightarrow C$. Elements of S^* are called __________

(A) Principal value distribution

(B) Finite part distribution

(C) Tempered Distributions

(D) Sinc Distributions

(Section-B)

(0.75 marks each)

Note- Answer the following very short answer type questions in 2-3 sentences each :

1. Write about Odd functions.

2. Write about Half Range Series.

3. State Uniqueness of Fourier Series.
4. Define Convolution of two functions.
5. Write the properties of fourier transform on Schwartz space.
6. Write about Shifting Property of Fourier Transform.
7. Define Schwartz Space.
8. Write one property of fourier transform on the Schwartz space.
9. Write about Polynomial growth.
10. Define Schwartz function.

(Section-C)

(1.25 marks each)

Note- Answer the following short answer type question in ≤ 75 words:

1. Find a fourier series to represent

   $f(x) = \pi - x \; for \; 0 < x < 2\pi$
2. Write about the fourier series for discontinuous functions.
3. Write about Abel means and summation.
4. Write about Mutually orthogonal set.
5. Write the Parseval's Identity for Fourier Transform.
6. State and prove Poisson Summation Formula.

7. If $f \in S(R)$, then prove that $(f * K_\delta)(x) \to f(x)$ uniformly in x as $\delta \to 0$.
8. State and prove Plancherel Theorem.
9. Write about Translator operator and Reflector operator.
10. Write about the application of Tempered distribution in partial differential equation.

(Section-D)

(2 marks each)

Note- Answer the following long answer type question using 175 words:

1. Find the fourier half range cosine series of the function

$$f(t) = \begin{cases} 2t, 0 < t < 1 \\ 2(2-t), 1 < t < 2 \end{cases}$$

OR

Obtain the fourier cosine series expansion of the periodic function defined by $f(t) = \sin\left(\frac{\pi t}{l}\right), 0 < t < l$

2. State and prove Fejer's Theorem?

OR

Write about a continuous function with divergent fourier series.

3. Derive the formula for Fourier Transform.

OR

Find the fourier transform of the function-

$$f(x) = \begin{cases} l + \frac{x}{a}, (-a < x < 0) \\ l - \frac{x}{a}, (0 < x < a) \\ 0, otherwise \end{cases}$$

4. State and prove Fourier Inversion Theorem.

OR

State and prove Riemann Lebesgue Lemma.

5. State and prove Convolution for Tempered Distribution.

OR

Write about the Fourier transform of tempered distributions.